EXPERIMENTAL STUDY OF TRANSIENT THERMOELECTRIC COOLING

II. EXTREMAL MODE OF CURRENT VARIATION

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The results are reported on an experimental study of transient thermoelectric cooling when through the thermoelement is passed a current which varies extremally. It is shown that the maximum cooling in the extremal mode is greater than the maximum cooling in the steady-state mode.

Earlier the authors have reported on an experimental study of transient thermoelectric cooling when a constant current $j > j_{opt}$ is passed through the thermoelement [1]. It has been shown there that in this case the maximum cooling ΔT_m does not exceed $\approx 0.7\Delta T^{ss}_{max}$ according to theory [1, 2].

It follows from the analysis in [3, 4] that considerable cooling can be effected when the current rises. Thus, if the current increases either according to a power law or exponentially, calculations will show a cooling of up to $\approx \Delta T^{ss}_{max}$ [5].

The maximum cooling, however, takes place theoretically [3] when the current j(t) varies "extremally" according to $A/\sqrt{t_0 - t}$ (A and t_0 are constants).

Here we present the results obtained in an experimental study of transient thermoelectric cooling with the current rising extremally.

An analysis of transient thermoelectric cooling [3, 6, 7, 8] shows that the basic laws followed by this mode of electronic cooling are governed by two effects: heat absorption at the cold junction (Peltier effect) and heat emission in the thermoelement bulk (Joule effect). According to the experimental evidence in [1], however, several basic peculiarities of transient thermoelectric cooling must be interpreted by taking into consideration an additional effect, namely extra Joule heat generation due to the contact resistance (R_c) in the cold junction.

When j = const, ΔT_m is much reduced on account of R_c [1]. This study will show that, when the current varies extremally, the emission of Joule heat due to the resistance in the cold junction can also seriously limit the extent of deep cooling predicted by theory [3].

Calculation of the Extremal Current Mode and of Transient Thermoelectric Cooling with a Contact Resistance in the Circuit. The following analysis is based on a model where the parameters Π, ρ, a, and $\varkappa$ of the substance are assumed independent of the temperature.

When the current varies with time, the general expression for cooling in the half-space ($at_0 \ll l^2$ [1], with l denoting the length of the thermoelement branch) can be written as (see Eq. (73) in [3]):

$$\Delta T(t_0) = \int_0^{t_0} \left[\frac{\Pi}{\varkappa} \sqrt{\frac{a}{\pi}} \cdot \frac{j(\tau)}{\sqrt{t_0 - \tau}} - \frac{\rho a}{\varkappa} j^2(\tau) \right] d\tau, \tag{1}$$

where t_0 is the pulse width. With resistance R_c in the circuit, Π becomes $\Pi - j(\tau)R_cS/2$ [1].

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Then

$$\Delta T(t_0) = \int_0^{t_0} \left[\frac{\Pi - \frac{1}{2} j(\tau) R_c S}{\varkappa} \sqrt{\frac{a}{\pi}} \cdot \frac{j(\tau)}{\sqrt{t_0 - \tau}} - \frac{\rho a}{\varkappa} j^2(\tau) \right] d\tau. \tag{2}$$

Differentiating (2) with respect to j and equating the derivative to zero, we find the extremal current mode, i.e., the mode of current variation which will yield the maximum cooling at the instant t_0:

$$j_{\mathrm{ext}}(\tau) = \frac{\Pi}{2\rho \sqrt{\pi a (t_0 - \tau)} + R_c S}. \tag{3}$$

When $R_c = 0$, expression (3) becomes

$$j_{\mathrm{ext}}(\tau) = \frac{\Pi}{2\rho \sqrt{\pi a (t_0 - \tau)}},$$

as has been derived in [3].

When $R_c \neq 0$, the maximum value of j is $j_m = \Pi / R_c S$ (at $t = t_0$), while $j \to \infty$ at $t \to t_0$ when $R_c = 0$. Inserting (3) into (2) and integrating, we obtain an expression for $\Delta T(t_0)$ which takes into account the contact resistance in the cold junction:

$$\Delta T(t_0) = \Delta T_{\max}^{\mathrm{ss}} \frac{1}{\pi} \ln \left(\frac{2\rho \sqrt{\pi a t_0}}{R_c S} + 1 \right). \tag{4}$$

Formula (4) indicates that the cooling by the extremal mode depends on the magnitude of R_c. As the contact resistance increases, ΔT decreases at a given t_0 and becomes zero at sufficiently large values of R_c. At a fixed R_c, the cooling increases as the pulse width t_0 increases.*

It can be shown without difficulty that $\Delta T(t_0) = \infty$ when $R_c = 0$.

An analogous result was obtained in [3], where $R_c = 0$ had been assumed. Taking R_c into account, according to (4), yields a finite ΔT.

It should be noted that the temperature drop at the cold junction is limited also by the temperature dependence of the Peltier coefficient ($\Pi = \alpha T \to 0$ as $T \to 0$) [3].

Let us evaluate the factor $\xi = \frac{1}{\pi} \ln\left(\frac{2\rho \sqrt{\pi a t_0}}{R_c S} + 1 \right)$ in (4). For typical values of ρ and a ($R_c S = 7.7 \cdot 10^{-4}\ \Omega \cdot \mathrm{cm}^2$ and $a = 10^{-2}\ \mathrm{cm}^2/\mathrm{sec}$) and the best value of $R_c S = 2 \cdot 10^{-5}\ \Omega \cdot \mathrm{cm}^2$ [9], $\xi = 1.53$ when $t_0 = 81$ sec and $\xi = 1.2$ when $t_0 = 9$ sec. For $R_c S = 2 \cdot 10^{-4}\ \Omega \cdot \mathrm{cm}^2$, $\xi = 0.83$ when $t_0 = 81$ sec and $\xi = 0.52$ when $t_0 = 9$ sec.

This evaluation shows that the extremal mode of transient thermoelectric cooling can yield much deeper cooling than the steady-state mode. On the other hand, this points to the necessity of reducing the contact resistance R_c which reduces the effectiveness of transient thermoelectric cooling by the extremal mode.

It is of interest how ΔT varies within the time interval from 0 to t_0. The final result in this case, derived from Eq. (1), is

$$\Delta T(t) = \Delta T_{\max}^{\mathrm{ss}} \frac{2}{\pi} \Bigg[\left(\ln \frac{\sqrt{t_0} + \sqrt{t}}{\sqrt{t_0 - t}} - \beta \Phi \right) - \frac{\beta}{2(1-\beta^2)} \left(\Phi - \frac{\beta \sqrt{t}}{\sqrt{t_0} + b} \right) - \frac{1}{2} \left(\ln \frac{\sqrt{t_0} + b}{\sqrt{t_0 - t} + b} + \frac{b}{\sqrt{t_0} + b} - \frac{\beta}{1 + \beta} \right) \Bigg], \tag{5}$$

where

$$b = \frac{R_c S}{2\rho \sqrt{\pi a}}, \quad \beta = \frac{b}{\sqrt{t_0 - t}}, \quad \Phi = \frac{2}{\sqrt{1-\beta^2}} \operatorname{arctg} \frac{K}{B} \quad \text{for } \beta < 1$$

* The magnitude of t_0 is limited by the stipulation that $a t_0 \ll l^2$ [1], which is easily shown to be equivalent to the stipulation that $j_0 \gtrless j_{\mathrm{opt}}$.

TABLE 1. Results of Tests Performed on Thermoelements

Elements	l, cm	S, cm^2	I_{opt}, A	ΔT_{max}^{ss}, °C	ΔT_m, °C	t_m, sec	ΔT_{ext}, °C	$\Delta T_{ext}/T_{max}^{ss}$
TE-17	2,9	0,28	7	60	46	55,5	78	1,3
TE-18	1,5	0,5	19	69	53	14,3	93	1,35
TE-19	6,5	2	21	66	53	240	77	1,17

Note: I_{opt} is the optimum thermoelement current, ΔT_{ext} is the maximum cooling produced by the extremal mode of current variation, ΔT_m is the maximum cooling produced by a constant current j = const. = $2j_{opt}$, t_m is the time to reach ΔT_m [1].

and

$$\Phi = \frac{1}{\sqrt{\beta^2 - 1}} \ln \frac{B + K}{B - K} \quad \text{for } \beta > 1,$$

$$K = \frac{\sqrt{t_0} - \sqrt{t_0 - t}}{\sqrt{t}}, \quad B = \sqrt{\left|\frac{1+\beta}{1-\beta}\right|}.$$

The first term inside the brackets represents Peltier heat absorption, the second term represents Joule heat emission in the junction, and the third term represents Joule heat emission in the thermoelement bulk.

When $R_c = 0$, formula (5) becomes

$$\Delta T(t) = \frac{\Pi^2}{2\pi\rho\varkappa} \ln \frac{(\sqrt{t_0} + \sqrt{t})^2}{\sqrt{t_0 (t_0 - t)}}.$$

RESULTS AND DISCUSSION

The parameters of the materials in the p- and n-branches of the thermoelements as well as the test procedure were analogous to those described in [1]. The thermoelectric effect, on the basis of the maximum temperature difference produced across a thermoelement in the optimum current mode, was $Z = 2.65 \cdot 10^{-3}$ deg^{-1}. Within the same temperature range the average value of Z for the materials of the thermoelement branches was $3 \cdot 10^{-3}$ deg^{-1}. The current of a complex pulse form was regulated with the aid of a specially designed rheostat system. Measurements were made with the initial temperature of the thermoelement branches 20°C. The temperature of the hot junction was held forcibly at 20°C (in separate tests at the end of the time interval we observed a heating of the hot junction, however, but by not more than 6-8°C). The vacuum was maintained at 10^{-4} torr maximum. In order to reduce the heat transfer from the ambient medium, the lateral surfaces of the thermoelements were additionally insulated. In order to reduce the temperature measurement error, the "bead" (junction) of the thermocouple was welded in between the thermoelement branches while the thermocouple itself had been covered with a thin coat of insulating varnish AK-400 and glued to the thermoelement all around the cold junction.

The essential test data for three thermoelements are given in Table 1. As can be seen here, the cooling by the extremal mode exceeds in all cases the cooling by the steady-state mode ($\Delta T_m < \Delta T_{max}^{ss} < \Delta T_{ext}$).

We will dwell on the test results for thermoelement TE-18. The cooling curves for this thermoelement (Fig. 1) have been recorded by means of a model N-004 light-beam oscillograph at t_0 = 9, 25, 36, 49, 64, and 81 sec. It is evident in Fig. 1 that, according to theory, ΔT_{ext} increases as t_0 is increased. Some reduction in ΔT_{ext} at t_0 = 81 sec occurs, apparently, because the hot junction begins to exert its influence (the condition of semiinfinity is disturbed) and some heat enters through the lateral surface (the problem conditions cease to be uniform). The $\Delta T(t)$ curve has been calculated according to Eq. (5) with given values of Π, ρ, $\varkappa$, a and with the directly measured value of R_cS ($5.4 \cdot 10^{-5}$ $\Omega \cdot$ cm^2). In Fig. 2 this curve is compared with the test curve. The calculated values of ΔT are lower than the test values almost throughout the entire interval and only in the end region (at t close to t_0) does the calculated curve rise above the test curve. Such a trend is explainable by the fact that the temperature dependence of the thermoelement parameters has not been considered in the calculations. In the initial cooling stage the true values of Π, ρ, and $\varkappa$ exceed their average values used in our calculations, while in the last stage they are smaller. On the same diagram we show the $\Delta T(t)$ curve for R_cS, which, not surprisingly, lies above the test curve.

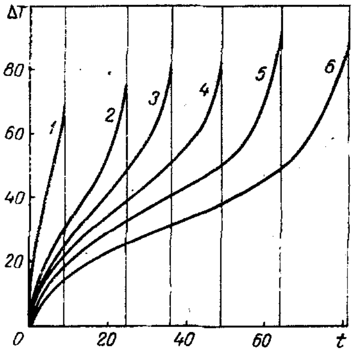

Fig. 1

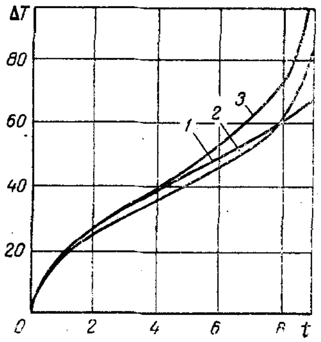

Fig. 2

Fig. 1. Curves of $\Delta T = f(t)$ for various values of t_0: 1) 9 sec; 2) 25 sec; 3) 36 sec; 4) 49 sec; 5) 64 sec; 6) 81 sec. ΔT, deg; t, sec.

Fig. 2. Comparison between the test curve (1) and the calculated curves (2, 3) of $\Delta T = f(t)$: 2) $R_c = 5.4 \cdot 10^{-5}\ \Omega \cdot cm^2$; 3) 0. ΔT, deg; t, sec; $t_0 = 9$ sec.

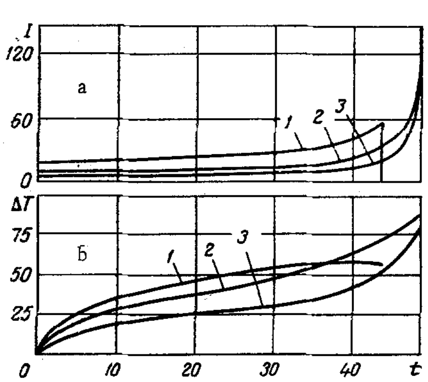

Fig. 3

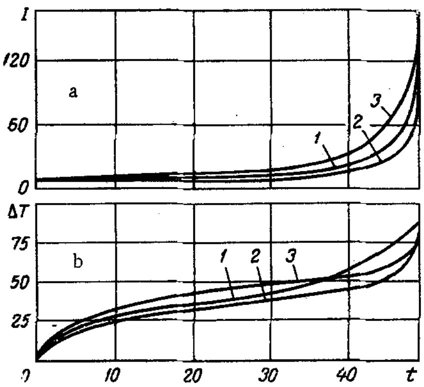

Fig. 4

Fig. 3. Transient thermoelectric cooling at various rates of current rise (a). $\Delta T = f(t)$ (b): 1) $\gamma = 1$; 2) 2; 3) 3. I, A; ΔT, °C; t, sec; $t_0 = 49$ sec.

Fig. 4. Effect of departure from the extremal mode of current variation on the magnitude of transient thermoelectric cooling ($t_0 = 49$ sec): a) j(t); ΔT(t): 1) $j = j_{ext}$; 2) $j < j_{ext}$; 3) $j > j_{ext}$. I, A; ΔT, °C; t, sec.

It must be noted that the extremal mode of current variation (3) has been established on the assumption that the branches of the thermoelement are identical, that the properties of their materials are independent of the temperature, that no heat enters through the lateral surface of a thermoelement, etc. In an actual case, otherwise mode (3) may cease, generally, to be the extremal one and may change slightly. Taking this into consideration, we performed two additional test series with a current mode more or less different than the calculated one.

In the first series (Fig. 3) the current was varied according to

$$j = \frac{\Pi}{\gamma\rho\sqrt{\pi a (t_0 - t)} + R_c S},$$

with γ denoting a numerical factor ($\gamma = 2$ corresponding to an extremal mode). The results shown in Fig. 3a, b were obtained in the first series with $\gamma = 1$, 2, and 3 ($t_0 = 49$ sec). A comparison of the $\Delta T(t)$ curves indicates a considerable cooling first (till t = 37 sec) along the $\gamma = 1$ curve and then, as the current increases, along the $\gamma = 2$ curve. After t = 42 sec, the cooling at $\gamma = 1$ decreased and the cold junction began to heat up. In order to avoid overheating the thermoelement, the $\gamma = 1$ current was turned off after t = 44 sec.

Along curve 3 the cooling ΔT increases toward the end of the time period, but remains all the time smaller than ΔT along curve 2.

The initial and the final values of the current in the second test series were the same as in the $\gamma = 2$ case, but inside the time interval the current was either larger (curve 3 in Fig. 4a) or smaller (curve 2 in Fig. 4a) than the current in the $\gamma = 2$ case. A comparison of the $\Delta T(t)$ curves in Fig. 4b indicates that in the beginning, with a larger current (curve 3), the transient thermoelectric cooling is greater. After $t = 36$ sec, however, cooling due to the extremal mode of current variation begins to predominate (curve 1). When $j < j_{ext}$ (curve 2), the cooling is smaller than in the $\gamma = 2$ case throughout the entire time period.

Similar tests with pulses of different width yielded analogous results.

Thus, even though a few factors had not been considered in the calculation, maximum cooling was obtained with a current varying according to (3).

According to the experiments, the cooling by the extremal mode is maximum ($\Delta T_{ext} = 93°C$) and 1.35 greater than ΔT^{ss}_{max} (at the same temperature of the hot junction). Moreover, the instant t_0 of reaching maximum cooling can be ascertained a priori.

According to the calculations, the necessary switching performance characteristics are more stringent for the extremal mode of cooling than for the steady-state mode.

Thus, the experimental study has shown that better cooling may be obtained by the extremal mode of current variation than by the steady-state mode and even then by means of two-stage thermoelectric batteries. The results obtained in the study of the extremal mode reveal new possibilities in thermal cooling.

NOTATION

α	is the coefficient of thermal emf;
ρ	is the resistivity;
$\varkappa$	is the thermal conductivity;
Π	is the Peltier coefficient;
a	is the thermal diffusivity;
l	is the length of thermoelement;
S	is the cross-section area of thermoelement;
j	is the current density;
j_{opt}	is the optimum current density;
j_{ext}	is the current density in the extremal mode;
j_0	is the initial current density;
R_c	is the contact resistance in the cold junction of a thermoelement;
t	is the time;
t_m	is the time to reach minimum temperature in the $j = \text{const}$ mode;
t_0	is the pulse width in the extremal mode;
ΔT^{ss}_{max}	is the maximum temperature difference in the steady-state mode;
ΔT_m	is the maximum cooling in the $j = \text{const}$ mode;
ΔT_{ext}	is the maximum cooling in the extremal mode.

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